# HARDY-LITTLEWOOD-POLYA INEQUALITY AND THE HADAMAR THREE-CIRCLE THEOREM

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The Hadamard three-circle theorem states that if $f(z)$ a holomorphic function on the annulus $r_1 \leq |z| \leq r_2$ and $M(r) = \max_{|z|=r} |f(z)|$, then $\log M(r)$ is a convex function of the $\log r$. The conclusion of the theorem can be restated as

$$M(r) \leq M(r_1)^{\frac{\log r_2/r}{\log r_2/r_1}} M(r_2)^{\frac{\log r/r_1}{\log r_2/r_1}}$$

for any three concentric circles of radii $r_1 < r < r_2$.

The Hardy-Littlewood-Polya inequality is the following one

$$\|x^{(k)}(\cdot)\|_{L_2(\mathbb{R})} \leq \|x(\cdot)\|_{L_2(\mathbb{R})}^{1-\frac{k}{r}} \|x^{(r)}(\cdot)\|_{L_2(\mathbb{R})}^{\frac{k}{r}}.$$

One may formulate it in the Hadamar three-circle theorem form. Namely, in the following form. $\log \|x^{(k)}(\cdot)\|_{L_2(\mathbb{R})}$ is a convex function of $k$. We use this fact to obtain optimal recovery methods for the $k$-th derivative on the basis of inaccurate information about some other derivatives.